STUDY OF LOW-COST ORBIT DETERMINATION SYSTEM FOR TETHERED SATELLITES

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ABSTRACT

The statistics of estimated errors when conducting the orbit determination of a tethered satellite system (TSS) by only using Doppler frequencies is shown. TSS consists of two satellites connected each other by a tether. The center of mass (CM) moves like a single satellite, so this system can be utilized in case of a single satellite as well as TSS. This is considered as a low-cost orbit determination system because it can be realized with simple instruments. This kind of low-cost ground station will charm institutions that are developing a micro satellite in a small budget. The result will provide a useful reference in constructing a ground station for the orbit determination by only Doppler frequencies.

INTRODUCTION

Satellites were being developed by specific special institutions until some years ago. However, development of micro satellites with low-cost concept has been recently increasing, which is developed by institutions such as universities1. These kinds of non-commercial satellites are very effective in high-risk missions and examinations of brand-new devices because the cost is not much expensive compared to the conventional ones.

In addition, a tethered satellite system (TSS) which consists of two satellites connected each other by a tether is an interesting topic in recent space systems, which applications cover a wide area2. In the category of micro satellites, TiPS (the Tether Physics and Survivability) was launched by the Naval Research Laboratory’s (NRL’s) Naval Center for Space Technology, and the missions were successful3. However, TSS has not been achieved to practical use and more experiments in orbit are necessary. Consequently, the experimental missions using low-cost micro tethered satellites will be expected to increase in future.

After the low-cost micro satellite is launched, however, the operation is conducted in a large-scale ground station under present conditions. It will become a burden if observations of satellites are all entrusted to such limited institutions. Therefore, if there is another orbit determination system that can be easily utilized in simple instruments, institutions that developed satellites will be able to determine the orbit by its own facilities as well as manage commands and telemetries.

Large-scale ground stations use high precision equipment and have the ability to determine satellites’ orbits in very high precision. The measurement sources are range, which can be obtained by such as SLR (Satellite Laser Ranging)4, range rate, azimuth, elevation, and so on.

The objective of this study is to show the statistics of estimated error when conducting the orbit determination of TSS only by Doppler frequencies. This is considered as a low-cost orbit determination system because it can be realized with simple instruments. The required equipment are amateur radio antennas, direction control motors, a frequency analyzer (spectrum analyzer) and a
personal computer to control these devices. In addition, the center of mass (CM) of TSS moves like a single satellite and error of CM is estimated, so this system can be utilized in case of a single satellite as well as TSS. About orbit determination of TSS, Ref. 5 and 6 are interesting and helpful to understand this study topic.

The Doppler frequency can be converted into a range rate, which is a change rate of distance between a ground station and a satellite. The orbit determination by only Doppler frequencies has lower precision than the case of additionally using a range and angles. Although it may not be utilized in some missions which require the orbit determination in high precision, the system will be very useful when only the knowledge on future passes are necessary to send commands or receive telemetries. Although there are also very low-cost ways such as GPS (Global Positioning System), the service is not guaranteed to continue permanently and some law problems for the space use.

Moreover, if similar facilities are placed in around the world, observed data in the same time span will increase and estimated error can be reduced. This supplements the demerit that a single ground station can only observe a partial orbital arc. Fortunately, instant data exchange is available by using the high-speed and low-cost Internet network. This kind of low-cost ground station will charm institutions that are developing a micro satellite in a small budget. The result will provide a useful reference in constructing a ground station for the orbit determination by only Doppler frequencies.

An orbit determination is that an estimated orbital state at epoch time is improved by extra observed data. The non-linear sequential batch least squares method is used and a somewhat precise orbit generator is necessary. The orbital state used in this paper includes the position and velocity of a center of mass, tether length, a mass ratio of main and sub satellites, angles and angular velocity of swing motion. The observed data are only Doppler frequencies. In this paper, it is shown how much error of the state at epoch is estimated in case of being given specific error covariance of measured data. At first, numerical simulations are conducted at an example orbit in some parameters. The theoretical background is concretely explained in Ref. 11, which includes a simple orbit generator of TSS model, transformation from state space to observed data, and transformation of estimated error covariance matrices. Ref. 7-9 are helpful to understand basic theories and numerical methods of orbital mechanics. Secondly, the precision of commercial frequency analyzer is experimentally measured by using an amateur radio station, and the effectiveness of the system is estimated.

**Kyushu University’s TSS Project**

In recent years, there have been put particular attention on developing micro-satellite systems weighing several dozen kilograms because of low cost involved in launching and operating such satellites. Considering these latest development and challenges involved, Kyushu University has undertaken the work of developing tether system using micro-satellites. The design and fabrication are conducted under the leadership of a university-level single laboratory. The orbit determination by using only Doppler frequencies is one of the missions. For reference, missions and concepts of this project are explained as private background.

The TSS mission is to be conducted in the following sequence:

1. As TSS is launched into the orbit, a boom is first deployed in order to achieve attitude stabilization of the system.
2. After having stable system, a sub satellite is deployed by using tether. The release of tether is carried out such that the velocity of sub satellite is controlled toward smooth arrival at the end of tether release and the system is still in stable motion.
3. The mission ends after the tether is cut by space debris.

With a view to reduce the system cost and increase its reliability, the following systems have
been considered:

- Tether Deployment Control System: The simple open loop tether deployment system is used in order to reduce complexity. The retrieval of tether is not considered.
- Boom System for Passive Attitude Control: Instead of attitude control system using thrusters or reaction wheels, a passive attitude control using tether and boom deployment has been considered.
- Low-cost Orbit Determination System: A sequential recording of an orbital state is necessary to measure the dynamics of TSS. It may be imagined that a commercial Global Positioning System is easily utilized, however, it has a protection to restrict a use for military and does not work in over some altitude. In addition, we will need to overcome some legal and cost difficulties when using GPS system. Therefore, a substitute low-cost orbital state observation system is necessary. The orbit determination system by using only Doppler frequencies is shown in this paper. Moreover, information of on-board sensors will be helpful to improve the precision.

About this project, Ref. 1 will be helpful. The image of TSS is shown in Fig. 1.

**NUMERICAL SIMULATIONS**

**Procedure**

The objective of simulations is that standard deviations of a TSS orbital state are estimated when the orbit determination using observed data of only Doppler frequencies is conducted. A TSS orbital state is defined as follows.

\[
X_{TSS} = \begin{bmatrix}
\Omega \\
i \\
\omega \\
a \\
e \\
\theta \\
\phi \\
\dot{\theta} \\
\dot{\phi} \\
m_x/m_t \\
l
\end{bmatrix}^T
\]  

(1)

\(\Omega, i, \omega, a, e, M\) are Keplerian orbital elements of a center of mass. \(\theta\) is an in-plane motion, \(\phi\) is an out-of-plane motion, \(m_x/m_t\) is a mass ratio, and \(l\) is a tether length. All variables are depicted in Fig. 2. In this dynamics model, following conditions are assumed.

- Both satellites are particles and a mass of tether is neglected
A tension force in tether is attracted toward a center of mass from each satellite and force is constant

No perturbation forces, and a length of tether is sufficiently small against an orbital radius

Constant tether length and no bending

The array of observed data is defined as follows.

\[ \mathbf{Z} = [\dot{\rho}_1 \quad \dot{\rho}_2 \quad \ldots \quad \dot{\rho}_n]^T \]  

\( \dot{\rho}_1, \dot{\rho}_2, \ldots, \dot{\rho}_n \) are range rates which can be easily transformed from Doppler frequencies. The each data is one of either satellite of TSS. The specific error standard deviation of observed Doppler frequencies can be assumed like \( \sigma_{f1}, \sigma_{f2}, \ldots, \sigma_{fn} \). The each deviation \( \sigma_{fi} \) is corresponds to one of a ground station that observe \( \dot{\rho}_i \), which is defined by operators of simulation. They are transformed into \( \sigma_{\rho1}, \sigma_{\rho2}, \ldots, \sigma_{\rho n} \), and these standard deviations give a covariance matrix as follows.

\[ \mathbf{R} = \begin{bmatrix} \sigma^2_{\dot{\rho}1} & 0 & \ldots & 0 \\ 0 & \sigma^2_{\dot{\rho}2} & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & \sigma^2_{\dot{\rho}n} \end{bmatrix} \]  

When the transformation function of \( \mathbf{X}_{TS} \rightarrow \mathbf{Z} \) is known, the sensitivity matrix \( \mathbf{H}_{TS \rightarrow Z} \) can be numerically calculated by the method of finite differencing\(^4\).

\[ \Delta \mathbf{Z} = \mathbf{H}_{TS \rightarrow Z} \Delta \mathbf{X}_{TS} \]  

The covariance matrix of \( \mathbf{X}_{TS} \) is defined as \( \mathbf{P}_{TS} \), which can be calculated from \( \mathbf{R}, \mathbf{H}_{TS \rightarrow Z} \) as follows.

\[ \mathbf{P}_{TS} = (\mathbf{H}_{TS \rightarrow Z}^T \mathbf{R}^{-1} \mathbf{H}_{TS \rightarrow Z})^{-1} = \begin{bmatrix} \sigma^2_{\Omega} & \sigma_i \sigma_{\Omega} & \ldots & \sigma_i \sigma_{i-1} \\ \sigma_i \sigma_{\Omega} & \sigma^2_i & \ldots & \sigma_i \sigma_{i-1} \\ \vdots & \ddots & \ddots & \vdots \\ \sigma_i \sigma_{i-1} & \ldots & \ldots & \sigma^2_{i-1} \end{bmatrix} \]  

Figure 2  TSS Model
The upper left 6 x 6 elements are ones of the center of mass, and lower right 6 x 6 elements are ones of the TSS local system. The covariance matrix of the center of mass is transformed to ones in other coordinate system as follows.

\[
P_{ECI} = H_{CLA \rightarrow ECI} P_{CLA} H_{CLA \rightarrow ECI}^T
\]

\[
P_{VCG(r)} = H_{ECI \rightarrow VCG} P_{ECI(r)} H_{ECI \rightarrow VCG}^T
\]

Where, CLA is classical Keplerian elements, ECI is earth centered inertia coordinate system, and VCG is vehicle centered coordinate system which principal axis toward ground station, which is depicted in Fig. 3. \(P_{CLA}\) is upper left 6 x 6 elements of \(P_{TS}\), and \(H_{CLA \rightarrow ECI}, H_{ECI \rightarrow VCG}\) can be numerically or analytically calculated as like \(H_{TS \rightarrow XYZ}\). The small \((r)\) means that it is including only covariance of positions. Where,

\[
X_{CLA} = \begin{bmatrix} \Omega & i & \omega & a & e \\ \end{bmatrix}^T
\]

\[
X_{ECI} = \begin{bmatrix} x & y & z & \dot{x} & \dot{y} & \dot{z} \\ \end{bmatrix}^T
\]

\[
X_{VCG} = \begin{bmatrix} x_v & y_v & z_v & \dot{x}_v & \dot{y}_v & \dot{z}_v \\ \end{bmatrix}^T
\]

\[
P_{VCG(r)} = \begin{bmatrix}
\sigma_{\dot{x}x}^2 & \sigma_{\dot{x}y} & \sigma_{\dot{x}z} & \sigma_{\dot{x}\dot{x}} & \sigma_{\dot{x}\dot{y}} & \sigma_{\dot{x}\dot{z}} \\
\sigma_{\dot{y}x} & \sigma_{\dot{y}y}^2 & \sigma_{\dot{y}z} & \sigma_{\dot{y}\dot{x}} & \sigma_{\dot{y}\dot{y}} & \sigma_{\dot{y}\dot{z}} \\
\sigma_{\dot{z}x} & \sigma_{\dot{z}y} & \sigma_{\dot{z}z}^2 & \sigma_{\dot{z}\dot{x}} & \sigma_{\dot{z}\dot{y}} & \sigma_{\dot{z}\dot{z}} \\
\end{bmatrix}
\]

Therefore, from \(P_{VCG(r)}\) and lower right 6 x 6 elements of \(P_{TS}\), following standard deviations can be obtained.

\[
\sigma_{\dot{x}x}, \sigma_{\dot{y}y}, \sigma_{\dot{z}z}, \sigma_{\dot{x}y}, \sigma_{\dot{x}z}, \sigma_{\dot{y}x}, \sigma_{\dot{x}x}, \sigma_{\dot{x}y}, \sigma_{\dot{x}z}, \sigma_{\dot{y}x}, \sigma_{\dot{y}y}, \sigma_{\dot{y}z}, \sigma_{\dot{z}x}, \sigma_{\dot{z}y}, \sigma_{\dot{z}z}
\]

**Simulation Parameters**

The assumed orbit is circle LEO (Low Earth Orbit) with 800-km altitude and 100-deg high inclination, which is typical one for science mission orbits. The epoch state of CM (Center of Mass) of TSS (Tethered Satellite System) is assumed as follows.

\[
\Omega_0 = 45.0 , \ i_0 = 100.0 , \ \omega_0 = 180.0 , \ a_0 = 800 \ km + 6378.14 \ km , \ e_0 = 0.001 , \ M_0 = 90.0
\]

The following common situations are assumed.
• The standard deviation of observed data is \( \sigma_{fd} = 1 \text{ Hz} \), \( f_0 = 435 \text{ MHz} \).
• Interval of measuring data: 2 data (one is from main satellite and the other is from sub satellite) every 2 min, that is 1 data/min.
• TSS is visible upper than 15° elevation
• Ground station: Fukuoka, Japan (130.429E, 33.622N, 5-m alt.)

Three kinds of simulations are conducted. The parameters for each simulation are defined in Table 1.

### Table 1
**SIMULATION PARAMETERS**

<table>
<thead>
<tr>
<th>Simulation No.</th>
<th>Span</th>
<th>( \theta_0 )</th>
<th>( \phi_0 )</th>
<th>( \theta_0 )</th>
<th>( \phi_0 )</th>
<th>( m_2 / m_1 )</th>
<th>( l )</th>
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<tr>
<td>1</td>
<td>0 – 7 days</td>
<td>0°</td>
<td>0°</td>
<td>0°</td>
<td>0°</td>
<td>1</td>
<td>1 km</td>
</tr>
<tr>
<td>2</td>
<td>3 days</td>
<td>0°</td>
<td>0°</td>
<td>0°</td>
<td>0°</td>
<td>1</td>
<td>1 – 100 km</td>
</tr>
<tr>
<td>3</td>
<td>3 days</td>
<td>0° – 60°</td>
<td>0°</td>
<td>0°</td>
<td>0°</td>
<td>1</td>
<td>1 km</td>
</tr>
</tbody>
</table>

**Simulation Results**

The results of simulations are shown in Figure 4 – 7.

From Sim. 1, \( \sigma_{v_x} \), \( \sigma_{v_y} \) are worse than others because Doppler frequencies does not reflect out-of-plane motion errors so much. The standard deviations become stable after 2 or 3 days, then monotonously decreasing. The degree of decreasing is simply derived by the number of data as follows.

\[
\sigma_n = \sqrt{\frac{m}{n}} \sigma_m
\]

Where, \( \sigma_m, \sigma_n \) are defined as estimated standard deviations derived by \( m \)-data and \( n \)-data in the same time span. In this simulation, only one ground station is used for observation. However, when using more ground stations, the number of data obtained in the same span will increase. The effect of using many ground stations is simply derived from the number of data. For example, when using the two ground stations, the number of data at the same span will be about twice, then standard deviations are improved like \( \sigma_n = \sqrt{1/2} \sigma_m \approx 0.7071 \sigma_m \).

From Sim. 2, When tether length becomes \( n \) times, standard deviations of swing angles becomes \( 1/n \) because standard deviations of each satellite’s position are constant in different values of the tether length. From Sim. 3, the larger epoch swing is, the smaller standard deviations of tether length and mass ratio become because large swing amplitude influences the sensitivity of the tether length and mass ratio effecting on observed data.

In these simulations, the standard deviation of observed data is assumed \( \sigma_{fd} = 1 \text{ Hz} \). However, the results can be easily converted into other cases by assuming that \( \sigma_{fd} \) is a random noise error. The relation between \( \sigma_{fd} \) and \( \sigma_i \) (\( i = x, y, z, \theta, \phi, m_2 / m_1, l \)) is constant. Therefore, when \( \sigma_{fd} \) is \( n \)-times, \( \sigma_i \) also becomes \( n \)-times.

\[
\frac{\sigma_i}{\sigma_{fd}} = \text{const}
\]
**Figure 4** Estimated Standard Deviation of CM and Swings in 7-day span (Sim. 1)

**Figure 5** Estimated Standard Deviation of Tether Length and Mass Ratio in 7-day span (Sim. 1)
Figure 6  Estimated standard deviations in different values of tether length at 3-day span (Sim. 2)

Figure 7  Estimated standard deviations in different values of epoch swing of theta at 3-day span (Sim. 3)
ERROR COVARIANCE OF COMMERCIAL FREQUENCY ANALYZER

Using the specification of following commercial spectrum analyzer, the simulations were examined:

- Advantest R3261CN: $10,000
- RBW (Resolution Band Width): 30 Hz
- VBW (Video Band Width): 10 Hz
- Sweep Frequency Span: 1000 Hz
- Minimum Sweep Time at Max Accuracy: 6.7 sec

The experiment of orbit determination was conducted to a real amateur satellite in orbit. From this experiment, standard deviation of measured frequency by this device was found to be about 50 Hz. Defining each estimated standard deviation shown in Simulation 1 is $\sigma_{mi}$, the value of $\sigma_{ni}$ by this device is calculated as follows:

$$\sigma_{ni} = \sqrt{\frac{1}{60/6.7}} \times 50 \times \sigma_{mi} = 16.7 \sigma_{mi}$$  \hspace{1cm} (14)

Therefore, estimated standard deviations when using this device are 16.7 times worse than results shown in Figure 4 and 5. The detail values are shown in Table 2. Moreover, the case when using 3 stations is examined. It is assumed that each ground station has same device. Defining each standard deviation of this case as $\sigma_{n3i}$,

$$\sigma_{n3i} = \sqrt{\frac{1}{3}} \sigma_{ni} = 9.67 \sigma_{mi}$$  \hspace{1cm} (15)

The results are also shown in Table 2. From this result, even when using this device in 3 stations, errors about tether swing motion are still large. Therefore, standard deviation of observed data is necessary to be much smaller than one of this device.

### Table 2

<table>
<thead>
<tr>
<th>Case</th>
<th>Simulation 1</th>
<th>Example by 1stn</th>
<th>Example by 3stns</th>
</tr>
</thead>
<tbody>
<tr>
<td>sigma_fd (Hz)</td>
<td>1.0</td>
<td>50.0</td>
<td>50.0</td>
</tr>
<tr>
<td>interval time (secs)</td>
<td>60.0</td>
<td>6.7</td>
<td>6.7</td>
</tr>
<tr>
<td>sigma_xv (m)</td>
<td>32.5</td>
<td>542.7</td>
<td>314.2</td>
</tr>
<tr>
<td>sigma_yv (m)</td>
<td>46.2</td>
<td>771.4</td>
<td>446.6</td>
</tr>
<tr>
<td>sigma_zv (m)</td>
<td>33.0</td>
<td>550.6</td>
<td>318.8</td>
</tr>
<tr>
<td>sigma_theta (deg)</td>
<td>2.3</td>
<td>39.1</td>
<td>22.7</td>
</tr>
<tr>
<td>sigma_phi (deg)</td>
<td>4.8</td>
<td>80.8</td>
<td>46.8</td>
</tr>
<tr>
<td>sigma_m2m1</td>
<td>0.2</td>
<td>3.8</td>
<td>2.2</td>
</tr>
<tr>
<td>sigma Ln (m)</td>
<td>91.2</td>
<td>1522.6</td>
<td>881.7</td>
</tr>
</tbody>
</table>

CONCLUSION

The objective of this study was to show the statistics of estimated errors when conducting the orbit determination of TSS (Tethered Satellite System) only by Doppler frequencies. In addition, this system can be utilized in case of a single satellite as well as TSS because CM (Center of Mass) of TSS moves like a single satellite. Although the simulations are conducted in parameters of 1-Hz standard deviation of analyzing frequency and 1-data/min measurement timing, the result can be converted into other cases. To estimate the performance of the low-cost orbit determination system, the estimated standard deviations of $\sigma_{xy}, \sigma_{yx}, \sigma_{xz}, \sigma_{yz}, \sigma_{m2/m1}, \sigma_{l}$ were used.
The supposed orbit is 800-km altitude circle LEO (Low Earth Orbit) with high inclination, which is typical one for science missions. The three simulations were carried out by using variable parameters of time span, tether length, and epoch in-plane swing angle. The results are shown in Fig. 4 – 7.

Moreover, error covariance of a commercial frequency analyzer was experimentally measured by chasing a real amateur satellite in orbit. The standard deviation of this device was about 50 Hz, and when using this value in simulations, estimated standard deviations of orbital state were obtained as like Table 2. The error is not small even when using 3 stations. The position errors are 314 m – 447 m and swing angle errors are 22 – 47 deg in 1-sigma. However, this result does not mean that this system cannot be effective. From this result, we can learn that we should use more precise devise to achieve some extent precision. After target of precision such as 10 m or 100 m is determined, then we can know how much precision of device is necessary by the simulation results.

However, using high precision equipment is opposite to the concept of this low-cost system. The cost will much increase in using high precision frequency analyzer. Therefore, some extra strategy is necessary for this system. As next step of this study, the case of using on-board sensors that measure position accelerations, angular accelerations, tether tension, and so on are studied. In addition, simulations of orbit determination along with mission schedule are necessary to estimate degree of convergence because the orbit determination does not always converge in every case.

REFERENCES